Oxford Cambridge and RSA

Thursday 19 June 2025 – Afternoon

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Time allowed: 2 hours

You must have:
- the Printed Answer Booklet
- the Insert
- a scientific or graphical calculator

QP

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v},\ \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)\big(f(x)\big)^n\,dx = \frac{1}{n+1}\big(f(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

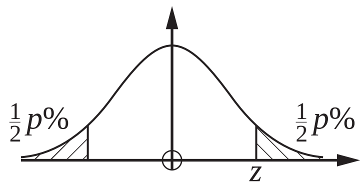
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A (60 marks)

1 **(a)** Find the first **three** terms in the binomial expansion of $(1+3x)^{\frac{1}{2}}$. **[3]**

(b) State the range of values of x for which this expansion is valid. **[1]**

2 You are given that

$\mathrm{f}(x) = \dfrac{3}{x}$ on the domain $\{x : x \in \mathbb{R}, x \neq 0\}$

$\mathrm{g}(x) = x - 1$ on the domain $\{x : x \in \mathbb{R}\}$.

Find $\mathrm{gf}(x)$. You must state the domain. **[2]**

3 The straight line with equation $2x + y = 6$ crosses the x-axis at A and the y-axis at B.

(a) Draw the line with equation $2x + y = 6$ on the grid in the Printed Answer Booklet. **[1]**

(b) Determine the equation of the perpendicular bisector of AB. **[6]**

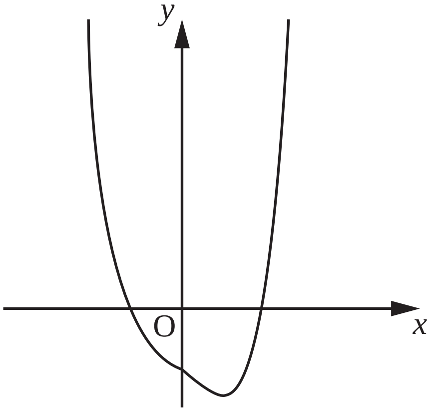
(c) Points A and B are opposite vertices of a square of side a.

Determine the exact value of a. **[3]**

4 The first term of a geometric sequence is 6.
The fourth term is $\frac{2}{9}$.
The sequence is infinite.

Find the sum of the associated series. **[3]**

5 The diagram shows the curve with equation $y = x^4 - 2^x$, for values of x close to zero.



(a) **In this question you must show detailed reasoning.**

Show that the equation $x^4 - 2^x = 0$ has a root which lies between 1 and 2. **[2]**

(b) Show that the equation $x^4 - 2^x = 0$ can be written in the form $x = 2^{0.25x}$ for positive values of x. **[1]**

(c) Use the iterative formula $x_{n+1} = 2^{0.25x_n}$ with $x_0 = 0.4$ to determine a root of $x^4 - 2^x = 0$ to **2** decimal places. **[2]**

(d) The diagram in the Printed Answer Booklet shows the line with equation $y = x$ and the curve with equation $y = 2^{0.25x}$.

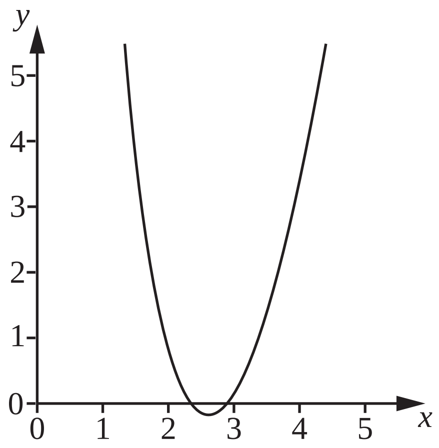
Sketch a cobweb or staircase diagram, on the diagram in the Printed Answer Booklet, for the iterative formula $x_{n+1} = 2^{0.25x_n}$ starting with $x_0 = 0.4$. Show at least **two** iterations. **[2]**

(e) Show that the equation $x^4 - 2^x = 0$ can be written in the form $x = 4\log_2 x$ for positive values of x. **[1]**

(f) **In this question you must show detailed reasoning.**

Show that the iteration $x_{n+1} = 4\log_2 x_n$, with $x_0 = 1$, will **not** find a root of $x^4 - 2^x = 0$. **[2]**

6 The diagram shows the curve with equation $y = f(x)$, where $f(x) = 13x - 34\ln(x) - 1.5$ for $x > 0$.



(a) A student attempts to locate a root of $f(x) = 0$ by using a spreadsheet to calculate the values of $f(x)$ for positive integer values of x.

Explain why the student's method may lead to the conclusion that $f(x) = 0$ has no roots. **[1]**

(b) Determine the exact value of the x-coordinate of the turning point of the curve. **[2]**

7 A group of scientists is studying how a population of insects grows in the laboratory.
At the start of the study, there are 48 insects.
At the end of the first month, there are 72 insects.
At the end of the second month, there are 108 insects.
The group develops two models, P and Q, to model the population.

The first model, P, is of the form $P_{t+1} = kP_t$, where k is a constant.
P_0 denotes the number of insects at the start of the study.
P_t denotes the number of insects at the end of month t, $t \geqslant 1$.
Model P fits the data for $t = 0$, 1 and 2 exactly.

(a) Find the value of k. **[1]**

(b) Use model P to predict the number of insects at the end of month 3. **[1]**

The second model, Q, is of the form $Q = 48\,e^{0.4t}$, where t is the time, in months, after the start of the study and Q denotes the number of insects at time t.

(c) (i) By considering the given data on the number of insects when $t = 1$ and $t = 2$, assess whether model Q fits the data for these times. **[2]**

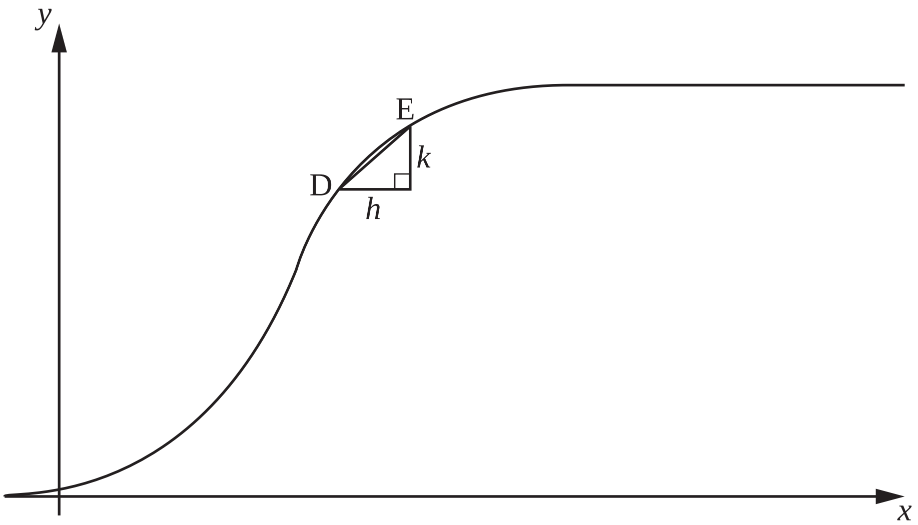
(ii) Show that model Q implies that the rate of growth of the population is proportional to the population at any time t. **[2]**

(iii) Show that, for integer values of t with $t > 1$, the population predicted by model Q will always be lower than the population predicted by model P. **[3]**

8 The diagram below shows the graph of y as a function of x.

Point D is the fixed point on the graph where $x = 10$.
Point E is the point on the graph where $x = 10 + h$ where h denotes a small increase in x.
The values of y at points D and E differ by k.

The value of the gradient of chord DE is $\frac{k}{h}$.



The screenshot below shows a spreadsheet used to find the gradient of DE for different values of h. The contents of some of the cells are not shown in the screenshot.

	A	B	C	D	E
1	*h*	10+*h*	*y*	*k*	gradient
2	1	11	1992.8	197.5937	197.594
3	0.1		1817.54	22.33784	
4	0.01	10.01	1797.46	2.259142	225.914
5	0.001	10.001	1795.43	0.226167	226.167
6	0.0001	10.0001	1795.22	0.022619	226.192
7	0.00001	10.00001	1795.2	0.002262	226.195
8					

(a) Write down the value for cell B3. **[1]**

(b) Find the value for cell E3. Give your answer to **3** decimal places. **[1]**

(c) Deduce, to **1** decimal place, the value of the gradient of the graph at D. **[1]**

9 The function $\mathrm{f}(x)$ is defined on all real numbers by $\mathrm{f}(x) = x^3 + \mathrm{e}^{3x}$.

(a) Differentiate $x^3 + \mathrm{e}^{3x}$. **[2]**

(b) Write down the coordinates of the point where the curve $y = \mathrm{f}(x)$ crosses the y-axis. **[1]**

(c) Find the gradient of the curve of the inverse function, $y = \mathrm{f}^{-1}(x)$, at the point where it crosses the x-axis. **[2]**

10 The magnitude of the vector $\begin{pmatrix} 4 \\ -1 \\ x \end{pmatrix}$ is an integer.

Determine all possible **integer** values of x. **[6]**

11 The nth term of a sequence is defined by $a_n = \dfrac{1}{\sqrt{n+1}-1} - \dfrac{1}{\sqrt{n+1}+1}$, for $n \geqslant 1$.

(a) Prove that every term in the sequence, a_n, is a rational number. **[3]**

(b) Determine whether the sequence is increasing, decreasing or neither. **[2]**

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

12 **(a)** Show that for the point on the curve where $x = 0$, $\dfrac{3a - x}{a + x} \geqslant 0$ as given in line 10. **[1]**

(b) Use the equation of the curve given in line 8 to show that for all points on the curve where $x \neq 0$, $\dfrac{3a - x}{a + x} \geqslant 0$ as given in line 10. **[1]**

(c) Show that it follows that $3a - x \geqslant 0$ and $a + x > 0$, as given in lines 10 and 11, for values of $a > 0$. **[3]**

(d) Hence find, in terms of a where $a > 0$, the range of values of x for points on the curve $y^2 = \dfrac{x^2(3a - x)}{a + x}$. **[1]**

(e) Find the equation of the asymptote to the curve. **[1]**

13 Use $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$, with suitable choices for A and B, to show that $\tan 3\theta = \dfrac{t(3 - t^2)}{1 - 3t^2}$, where $t = \tan\theta$, as given in line 25. **[3]**

14 **(a)** Draw and label a suitable triangle on the diagram in the **Printed Answer Booklet** to show that $\tan\theta = \dfrac{y}{x}$, as given in line 30. **[1]**

(b) Subtract $x = \dfrac{a(3 - t^2)}{1 + t^2}$ from $3a$ to show that $3a - x = \dfrac{4at^2}{1 + t^2}$. **[1]**

(c) Find an expression, in terms of a and t, for $a + x$. **[1]**

(d) Hence, or otherwise, show that the parametric equations given in lines 29 and 31 are equivalent to $y^2 = \dfrac{x^2(3a - x)}{a + x}$, as claimed in lines 8 and 33. **[2]**

END OF QUESTION PAPER

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